An experimental investigation of asymptotic hypersonic flows

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Hypersonic flow past axisymmetric power-law bodies ($y \propto x^m$; $m = 0, \frac{1}{10}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$ and 1) has been studied experimentally and a detailed analysis of the dependence of shock-wave shape on body shape made.

Results indicate a continuous relationship between the exponent of the shock-wave shape and that of body shape over the range considered.

1. Introduction

In a recent paper (Freeman 1962), the factors influencing the flow past blunt bodies at hypersonic speeds were discussed for the region far away from the blunt nose. These flows were called 'asymptotic flows' since one method of analysing their structure is to attempt to construct theoretically an asymptotic expansion of the analytic solution. Consideration of the nature of the flows at large distances from blunt bodies at hypersonic speed is by no means new (see, for example, the references contained in Freeman 1962) and many theoretical approaches to obtaining solutions have been made. Yakura (1962) has recently investigated the flow behind certain analytic shock shapes and draws conclusions which appear to contradict some of the basic tenets of the earlier work. It has become imperative, therefore, to examine in detail the nature of the mathematical expansion procedure which, it has been assumed, could be constructed from a first-order theory derived mainly from physical arguments.

On reviewing the available literature it is obvious that there are many apparent anomalies in the theories as presented. Although, for example, it is well known that similarity solutions do exist, it is not obvious why they exist only for a limited range of body shapes. Also, there seems to be a tendency in the literature to perpetuate a belief that the error, which is introduced by assuming that the flow field for a blunt body is similar to that given by hypersonic small-disturbance theory, must be the same as that for the small disturbance theory when applied to slender bodies, viz. of order the inverse square of the Mach number. The part played by the nose bluntness in determining the form of the asymptotic flow depends on the asymptotic shape of the body. If the asymptotic growth is sufficiently rapid, it will itself determine the first-order asymptotic flow field and the nose bluntness then introduces only an entropy layer as a second-order effect. Mathematically, this means that the asymptotic flow field is determined by the surface boundary condition and the nose bluntness introduces an error term which is small over the major part of the field. If, however, the growth of

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the body is slow, then the bluntness of the nose dominates even the asymptotic flow field, and, mathematically, the first term of the expansion is given by the blast-wave solution for a point source of constant energy.

For more details of the theoretical solution of the problem the reader is referred to the review and analysis given in Freeman (1962). In order to substantiate the method of expansion suggested it is strictly necessary to answer many questions as to the convergence and existence of the iterative procedure employed. The complexity of the problem would, at present, seem to make such considerations difficult. A situation of this type is, of course, not new in gas dynamics. One method of approach which can be applied in such cases is to verify some aspect of the theory by experimental measurements in order to show that at least the first terms in the expansion are correct.

Some of the limitations on the experimental confirmation of the theory will be discussed below. The apparent simplicity of some of the foundations for the theory, however, would seem to make it worth while to design an experiment which could be analysed in such a way as to test one of the basic assumptions of the theory.

The simplest experimental technique is to photograph the shock-wave shape on a variety of bodies. Although the theory is essentially only concerned with asymptotic flows the grosser features of the nose-geometry of the body are shown to influence the results. The most satisfactory method of selecting bodies which grow asymptotically as a power of the distance is to take the complete power-law shapes themselves. These bodies will be blunt provided that the power is less than unity.

The theory states that, asymptotically, for bodies of the form $y \propto x^{m_b}$, the shock is given by $y \propto x^{m_s}$ where

$$m_s = m_b \quad \text{for} \quad \tfrac{1}{2} < m_b \leqslant 1, \tag{1}$$

$$m_s = k \quad \text{for} \quad 0 \leqslant m_b < \tfrac{1}{2}, \tag{2}$$

where $k = 2/(3+j)$ with $j = 0$ for plane flow and $j = 1$ for axisymmetric flow. Since we shall only be concerned with axisymmetric flow, $k = \frac{1}{2}$. In case 1, similarity solutions in the form suggested by Lees & Kubota (1957) are available and in case 2 the solution is given by 'blast-wave analogy' as postulated by Lees (1956) and Cheng & Pallone (1956).

The author is not aware that tests have been made to verify the above hypothesis except for the original experiments of Kubota made to check case 1. The tests to be described in the present paper were carried out in the N.P.L. 6 in. shock tunnel. A series of models were constructed to cover the whole range of $0 \leqslant m \leqslant 1$ in the axisymmetric case. The models were complete power-law shapes with $m_b = 0, \frac{1}{10}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$ and 1. Their length (l) was in all cases 6 in. and their base diameter ($2r$) 2 in.

2. Results

Shadowgraphs of the flow past the model were taken at a flow Mach number (M_∞) of 8·8 and a Reynolds number of 2×10^6 per ft. The shock-wave shape was then obtained directly by constructing a system of ordinates on an enlarged photographic print. The shock-wave shapes obtained from the various bodies are plotted in figure 2 and a series of typical photographs shown in figure 1, plate 1. A curve of shock shape for the condition $m_b = 0$ taken from the Princeton helium tunnel results for $M_\infty = 12$, is also given on figure 2.

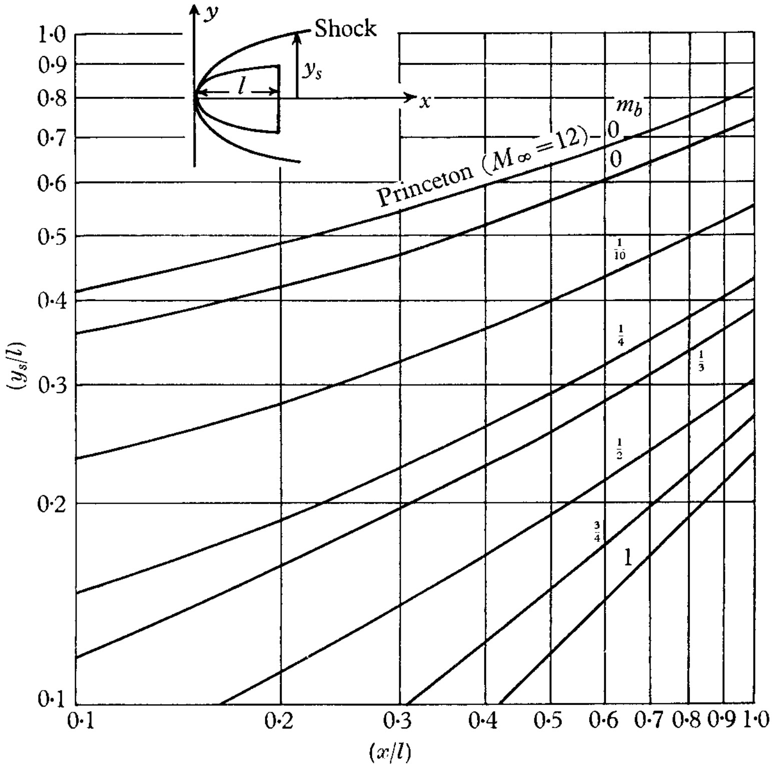

FIGURE 2. Shock-wave shapes ($M_\infty = 8{\cdot}8$).

Since figure 2 is plotted logarithmically, the slope of the curves gives the exponent (m_s) directly. It is apparent that over the range shown there is a considerable variation of the exponent with downstream distance on all the curves except for the cone ($m_b = 1$).

To compare these results with those predicted by the theoretical model it is necessary to eliminate the effect of the higher-order terms in the expansion.

These terms originate from two main assumptions. The analogy between two-dimensional steady flow and one-dimensional unsteady flow will only be a useful one if M_∞^{-2} is sufficiently small. This requirement can, in general, be easily met. A more stringent condition is imposed by the error terms introduced by the region near the body where the first-order solution is not uniformly convergent. An analysis of the flow in this neighbourhood (Freeman 1962) shows that these terms will be of order $(M_\infty^{-2})^\alpha$ where

$$\alpha = \frac{m_b}{1-m_b} - \frac{1}{\gamma} \quad \text{for} \quad \frac{1+\gamma}{2\gamma+1} > m_b > \tfrac{1}{2}, \tag{3}$$

$$\alpha = 1 - 2m_b \quad \text{for} \quad \tfrac{1}{2} > m_b > 1/2\gamma, \tag{4}$$

and

$$\alpha = 1 - 1/\gamma \quad \text{for} \quad 1/2\gamma > m_b > 0. \tag{5}$$

Relations (3) and (5) are due to entropy-layer effects and the relation (4) to body displacement. As will be seen this error can become quite large. In particular for the case $m_b = \frac{1}{2}$, it becomes dominant—a sign presumably that the similarity analysis is no longer adequate.

In order to remain within the domain where similarity solutions are available we require that $x' = (x/l)\, M_\infty^{-1/(1-m_s)}$ be small, with, at the same time, x/l large to preserve the asymptotic nature of the solution. This criterion is difficult

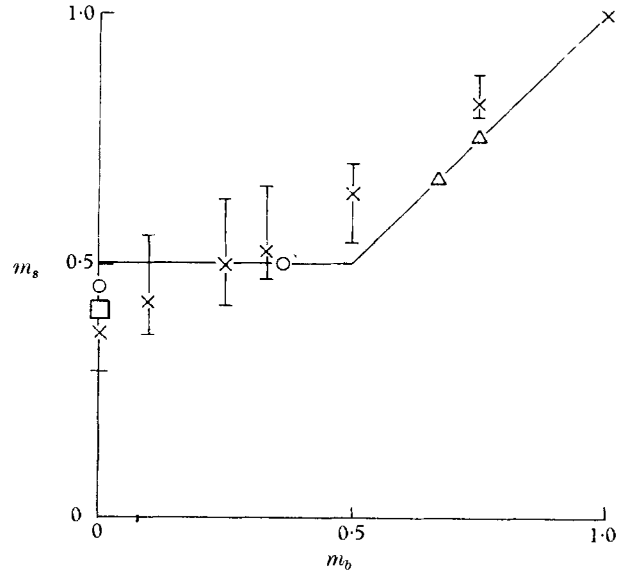

FIGURE 3. Shock exponent m_s plotted against body exponent m_b. +, Experiment ($M_\infty = 8{\cdot}8$, nitrogen); ○, Yakura (theoretical); ———, theory; □, Princeton ($M_\infty = 12$, helium); △, Kubota ($M_\infty = 7{\cdot}7$, air); I, variation between $x/l = 0{\cdot}2$ and $1{\cdot}0$.

to satisfy since if m_s is to be evaluated from the experimental results at a fixed x' it will be obvious that the actual position of measurement x must vary for different bodies (i.e. values of m_b). This variation can be very large indeed. For example, with tests at $M_\infty = 8{\cdot}8$, the measurement must be made at a distance almost eighty times as far downstream for a body with $m_b = \frac{3}{4}$ as for one with $m_b = \frac{1}{2}$. This is almost impossible to achieve experimentally.

Some compromise must be made in assessing these results. It was decided therefore to choose a particular value of x/l which is only strictly adequate where m_s does not vary with m_b (equation (2)) and measure the slope from figure 2. These are plotted for $x/l = 0{\cdot}5$ in figure 3 and the magnitude of the variation over this range $0{\cdot}2 < x/l < 1{\cdot}0$ is indicated. As might be expected the points fall on a smooth curve. The other points inserted are due to Kubota (1957) who measured the exponent for $m_b = \frac{2}{3}$ and $\frac{3}{4}$ to confirm the similarity solutions of Lees & Kubota (1957). Although the agreement was stated to be satisfactory, a close examination of the experimental results indicates that the slope has to be chosen judiciously to achieve this agreement and some deviation is obtained—although, in fact, it seems somewhat less than that obtained in the present tests. Kubota's bodies were of length-to-diameter ratio 1 and 2 compared with a value of 3 in the present tests. The models used by Kubota are thus not so slender as the N.P.L. ones. A further result plotted is that obtained at $M_\infty = 12$ in the Princeton helium tunnel for $m_b = 0$. The present review of asymptotic hypersonic flows (Freeman 1962) was originally initiated to investigate some results obtained by Yakura (1962) when considering the analytic solution of the inverse problem, which assumes a given shock-wave shape. Yakura obtained the body shape required for flow through a hyperboloidal shock wave ($m_s = 1$) and a paraboloidal shock ($m_s = \frac{1}{2}$). These points are plotted on figure 3. For $m_s = \frac{1}{2}$, an asymptotic body shape with $m_b = 1/2\gamma$ was obtained. This led Yakura to conjecture that for $m_b = 0$, m_s should be 0·46 as indicated in figure 3. No suggestion is made as to the type of behaviour to be expected between these values.

It is unfortunate that the experimental evidence is not able to distinguish between the two theories. It seems possible, however, that where the theory gives a large error term a slower convergence of the expansion would result and a consequent modification of the parameters in a similarity analysis would occur. These would almost certainly tend to smooth out the discrete changes suggested by theory. If such is the case, then it will probably be impossible to use experimental results to elucidate the theoretical approach. The difference between specifying an analytic shock shape or an analytic body shape may mean that a radical change is required in the mathematical expansion but need not necessarily mean that the final numerical values are greatly different. This would account for the smoothness of the variation with m_b of the experimental results which do not appear to be very sensitive to small perturbations of body shape (i.e. of m_b).

3. Experimental errors

Aside from the difficulties of comparison between theory and experiment discussed in § 2, there are obvious errors introduced by the experimental procedure itself. The tests were made in a conical nozzle and hence Mach-number gradients would be obtained along the model. These are known to be quite small, however. The Mach number remains within 1 % of its stated value throughout the region where measurements have been made. The extremely good conical shocks obtained on the cone confirms this. Boundary-layer growth on models of this length could also produce a displacement effect which would tend to distort the actual body shape and make it difficult to compare with a power-law theory. This effect is also very small as was confirmed by repeating the tests at a higher Reynolds number. Increasing the Reynolds number by a factor of two produced an effect hardly discernable on figure 2.

Refraction effects, causing an apparent displacement of the shock wave relative to the model, would also occur due to the use of the shadowgraph technique although these might be expected to be quite small since the bodies are axisymmetric. The maximum absolute error in determining the shock-wave ordinate from a photograph is of the order of 2 %. Since, however, the exponent only is required, the relative error between points, which would be considerably smaller than this, is more relevant. This can be minimized by measuring to a particular part of the finite-thickness image of the shock wave on the photograph.

4. Conclusions

The experiments show, that, analysed on a similarity basis, the shock-wave shapes change uniformly with m over the range $0 \leqslant m \leqslant 1$. This result might have been expected, since any abrupt changes required by the theory might be expected to have been smoothed out by the slower convergence of the expansion procedure in this neighbourhood. Although the experimental results indicate the basic trend suggested by theory, they do not allow any quantitative information, such as might be required to differentiate between two theories, to be deduced. It would appear that calculation of the higher-order terms in the expansion procedure would be necessary before any closer correlation of theory and experiment could be usefully made. As has been stated elsewhere, this is a formidable task but one which would greatly enhance our understanding of the factors influencing the development of hypersonic flows.

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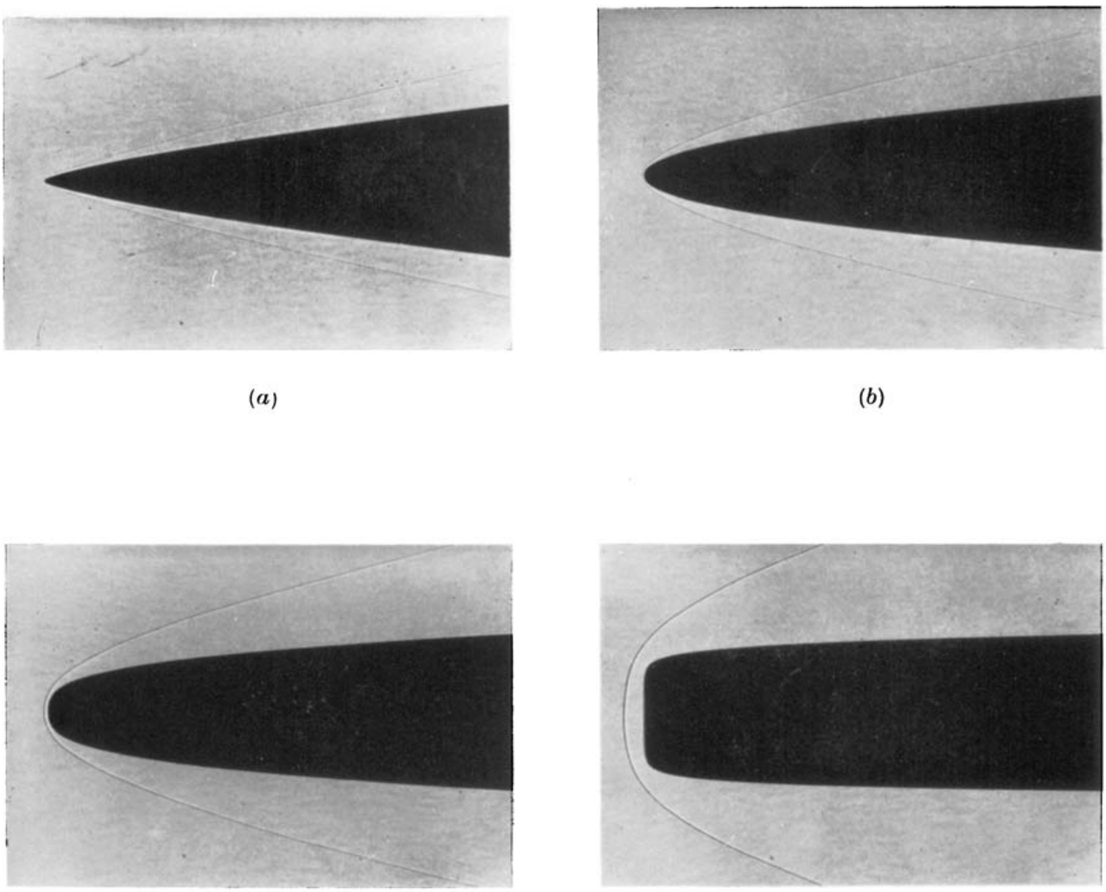

(*a*) (*b*)

(*c*) (*d*)

FIGURE 1. Shadowgraphs of shock-wave shape for bodies $y \propto x^{m_b}$ ($M_\infty = 8{\cdot}8$)
(*a*) $m_b = \frac{3}{4}$, (*b*) $m_b = \frac{1}{2}$, (*c*) $m_b = \frac{1}{3}$, (*d*) $m_b = \frac{1}{10}$.